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Magnetoelastic interaction between a soft ferromagnetic elastic half-plane with a crack and a constant magnetic field

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Abstract

The stress state of a magnetoelastic half-plane with a crack is considered. It is assumed that the half-plane is located in a magnetic field, which is parallel to the boundary of the half-plane. Four various boundary conditions for the half-plane are considered. A numerical method is developed to determine the crack-opening displacements and the magnetoelastic stress intensity factor. The effect of the magnetic field and the boundary conditions on the magnetoelastic stress intensity factor are shown graphically and numerically. The case of an edge crack is considered as a particular case.

Keywords: Magnetoelastic; Crack

1. Introduction

During last few years a great number of papers are devoted to the behavior of the deformable bodies in electromagnetic fields (Moon, 1968; Parton and Kudriavcev, 1988). Particularly the investigation of the stress state of magnetoelastic bodies with different types of defects (cracks, etc.) in magnetic fields becomes of great value (Parton and Kudriavcev, 1988; Shindo, 1977). Shindo (1977, 1982, 1983) was the first to consider the problems of determination of the stress–strain state of a ferromagnetic body with cracks in a magnetic field. He has proved, that quite weak magnetic fields (~1 T) can essentially change the stress–strain state of the body in the vicinity of the crack. Later on the influence of various factors (heterogeneity, nonstationarity of the process, account of the bound areas of the body etc.) on stress–

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strain state of a magnetoelastic body with a crack, has been considered (Hasanian et al., 1988; Hasanian and Bagdasaryan, 1997; Shindo, 1982, 1983). All mentioned problems are solved on the basis of linearized theory of magnetoelasticity of ferromagnetic bodies suggested by Pao and Yeh (1973).

In the present work the influence of boundary conditions on the stress–strain state of a half-plane with a crack is investigated. A case is considered when the crack is perpendicular to the boundary of the half-plane. The body is located in a homogeneous magnetic field, which is parallel to the boundary of the half-plane (Fig. 1). A singular integral equation is obtained with respect to the unknown function characterizing the problem. The problem is solved for the following boundary conditions:

- The boundary of the half-plane is fastened;
- The boundary of the half-plane is free of stresses;
- Mixed boundary conditions for the half-plane are given.

There are many papers devoted to similar questions in absence of the magnetic field (pure elastic case). Particularly, Koiter (1965), Nied (1987) and Panasuk et al. (1976) have considered the stress–strain state of the half-plane with a crack in the case when the boundary of the half-plane is free of stresses. There are many results obtained for the problem of a half-plane with a fastened boundary (Bereznicki et al., 1983; Savruk, 1988).

2. Formulation of the problem

Let the isotropic, homogeneous, linear elastic, magnetosoft ferromagnetic half-plane with a crack of width $l = b - a$ be located in a magnetic field $\mathbf{B} = (0, B_0)$, where $B_0 = \text{const}$. The cartesian coordinate system is chosen in such a way that the cross-section of the crack is in the plane X_1OX_2 and includes the segment (a, b) of the co-ordinate axis OX_1 ($\Omega_e = \{a < x_1 < b; x_2 = 0\}$). The boundary of the half-plane coincides with the axis OX_2 (see Fig. 1). The domain $\Omega_1 = \{-\infty < x_1 < 0; |x_2| < \infty\}$, that is denoted by (I) in Fig. 1, is taken to be vacuum. Let $\Omega_2 = \{0 < x_1 < \infty; |x_2| < \infty\}$ and $\Omega = \Omega_2/\Omega_e$ be the ferromagnetic body. The linearized equations and boundary conditions of magnetoelasticity for the considered problem according to the theory of Pao and Yeh (1973) are given in the following form: In the domain Ω

$$\Delta U_i + \frac{1}{1-2\nu}\left(U_{1,\,1} + U_{2,\,2}\right)_{,i} + \frac{2\chi b_c^2}{\mu_r}\varphi_{,i2} = 0 \quad (i = 1, 2) \tag{1}$$

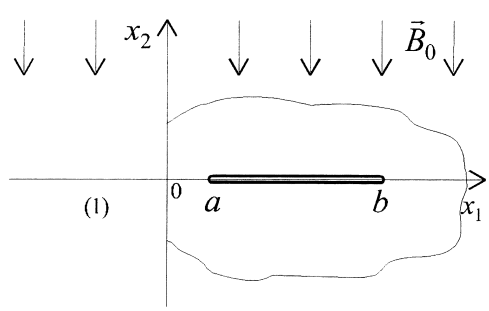

Fig. 1. Half-plane with a crack in a magnetic field.

$$\Delta\varphi = 0, \quad \mathbf{h} = \frac{B_0}{\mu_0}\,\mathrm{grad}\,\varphi \tag{2}$$

in the domain Ω_e

$$\Delta\varphi^{(e)} = 0, \quad \mathbf{h}^{(e)} = \frac{B_0}{\mu_0}\,\mathrm{grad}\,\varphi^{(e)} \tag{3}$$

in the domain Ω_1

$$\Delta\varphi^{(1)} = 0, \quad \mathbf{h}^{(1)} = \frac{B_0}{\mu_0}\,\mathrm{grad}\,\varphi^{(1)} \tag{4}$$

The boundary conditions on the line $x_2 = 0$ have the following form:

$$t_{12}(x_1, 0) = 0 \quad \text{where } 0 < x_1 < \infty, \tag{5}$$

$$\frac{t_{22}(x_1, 0)}{\mu} = \frac{\chi^2 b_c^2}{2\mu_r^2} + \frac{\chi^2}{\mu_r} b_c^2 \varphi_{,2} - P_0(x_1) \quad \text{where } a < x_1 < b, \tag{6}$$

$$\varphi_{,1}^{(e)} - \varphi_{,1} = -\frac{\chi}{\mu_r} U_{2,\,1} \quad \text{where } a < x_1 < b, \tag{7}$$

$$\varphi_{,2}^{(e)} - \mu_r \varphi_{,2} = 0 \quad \text{where } a < x_1 < b, \tag{8}$$

$$\varphi_{,1} = 0 \quad \text{where } 0 < x_1 < a, \text{ and } b < x_1 < \infty, \tag{9}$$

$$U_{2,\,1} = 0 \quad \text{where } 0 < x_1 < a, \text{ and } b < x_1 < \infty. \tag{10}$$

The relation (6) holds under the condition that the crack surface is subjected also to equal and symmetric mechanical loading, specified by $\mu \cdot P_0(x_1)$. In Eqs. (1)–(10) the following notations are accepted: $b_c^2 = B_0^2/\mu_0\mu$; μ, ν are the elastic constants; $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$; $f_{,i} = \frac{\partial f}{\partial x_i}$; $U_i(x_1, x_2)$ $(i = 1, 2)$ are the displacements of the media; $\mathbf{h}$, $\mathbf{h}^{(e)}$ and $\mathbf{h}^{(1)}$ are perturbed magnetic fields in domains Ω, Ω_e, and Ω_1 respectively; φ is the magnetic potential; μ_0 is the universal magnetic constant; $\chi = \mu_r - 1$ is the magnetic susceptibility.

Consider the following four boundary conditions on the line $x_1 = 0$ for $|x_2| < \infty$

$$\text{(A) } U_1(0, x_2) = 0,\ U_2(0, x_2) = 0;\ \varphi_{,2} - \varphi_{,2}^{(1)} = 0;\ \mu_r\varphi_{,1} - \varphi_{,1}^{(1)} = 0 \tag{11a}$$

$$\text{(B) } t_{11}(0, x_2) = 0;\ t_{12}(0, x_2) = 0;\ \varphi_{,2} - \varphi_{,2}^{(1)} = 0;\ \mu_r\varphi_{,1} - \varphi_{,1}^{(1)} - \chi U_{1,\,2} = 0 \tag{11b}$$

$$\text{(C) } U_2(0, x_2) = 0;\ t_{11}(0, x_2) = 0;\ \varphi_{,2} - \varphi_{,2}^{(1)} = 0;\ \mu_r\varphi_{,1} - \varphi_{,1}^{(1)} - \chi U_{1,\,2} = 0 \tag{11c}$$

$$\text{(D) } U_1(0, x_2) = 0;\ t_{12}(0, x_2) = 0;\ \varphi_{,2} - \varphi_{,2}^{(1)} = 0;\ \mu_r\varphi_{,1} - \varphi_{,1}^{(1)} = 0 \tag{11d}$$

In Eqs. (5)–(11d) the following relations are used (Pao and Yeh, 1973)

$$\frac{t_{ij}}{\mu}=\frac{\sigma_{ij}}{\mu}+\frac{\chi B_{0i}B_{0j}}{\mu_0\mu_r^2\mu}+\frac{\chi}{\mu_r\mu}\left[B_{0i}h_j+B_{0j}h_i\right];\ \frac{\sigma_{ij}}{\mu}=U_{i,\,j}+U_{j,\,i}+\frac{2\nu}{1-2\nu}\delta_{ij}U_{k,\,k}$$

$$\frac{t_{ij}^M}{\mu}=\frac{B_{0i}B_{0j}}{\mu\mu_0\mu_r}-\frac{B_{0k}^2}{2\mu\mu_0\mu_r}\delta_{ij}+\frac{B_{0i}h_j+B_{0j}h_i}{\mu}-\frac{B_{0k}h_k}{\mu\mu_r}\delta_{ij} \tag{12}$$

Besides the conditions (5)–(11d), the conditions at the infinity has to be taken into account, according to which all the unknown functions conditioned by the deformation of the media, tend to zero, for $x_1^2+x_2^2\to\infty$. Note that, the given equations and boundary conditions are written for the plane deformation state. By similarity, one can write down the equations and the boundary conditions for the plane stress state.

The formulation of the problem in this form is symmetric with respect to the axis OX_1. Using the Fourier integral transformation and the symmetry of the problem it can be shown that the solutions, satisfying Eqs. (1)–(4), have the following form: in the internal domain ($(x_1,x_2)\in\Omega$)

$$U_1(x_1,x_2)=\frac{2}{\pi}\int_0^\infty\left[E(\beta)+x_1F(\beta)\right]e^{-\beta x_1}\cos\beta x_2\,d\beta+\frac{2}{\pi}\int_0^\infty\left[A(\alpha)+(\alpha x_2-3+4\nu)\frac{B(\alpha)}{\alpha}+(1-2\nu)\frac{2\chi b_c^2}{\mu_r}C_1(\alpha)\right]e^{-\alpha x_2}\sin\alpha x_1\,d\alpha$$

$$U_2(x_1,x_2)=\frac{2}{\pi}\int_0^\infty\left[A(\alpha)+x_2B(\alpha)\right]e^{-\alpha x_1}\cos\alpha x_2\,d\alpha+\frac{2}{\pi}\int_0^\infty\left[E(\beta)+(\beta x_1-3+4\nu)\frac{F(\beta)}{\beta}-(1-2\nu)\frac{2\chi b_c^2}{\mu_r}C_2(\beta)\right]e^{-\beta x_2}\sin\beta x_1\,d\beta$$

$$\varphi(x_1,x_2)=\frac{2}{\pi}\int_0^\infty C_1(\alpha)e^{-\alpha x_2}\cos\alpha x_2\,d\alpha+\frac{2}{\pi}\int_0^\infty C_2(\beta)e^{-\beta x_2}\,d\beta \tag{13}$$

in the external domain, i.e. $(x_1,x_2)\in\Omega_1$

$$\varphi^{(1)}(x_1,x_2)=\frac{2}{\pi}\int_0^\infty G(\beta)e^{\beta x_1}\sin\beta x_2\,d\beta \tag{14}$$

in the domain $(x_1,x_2)\in\Omega_e$

$$\varphi^{(e)}(x_1,x_2)=\frac{2}{\pi}\int_0^\infty A_e(\alpha)\sin\alpha x_2\cos\alpha x_1\,d\alpha. \tag{15}$$

In Eqs. (13)–(15) $A(\alpha)$, $B(\alpha)$, $C_1(\alpha)$, $C_2(\alpha)$, $E(\beta)$, $F(\beta)$, $G(\beta)$ and $A_e(\alpha)$ are unknown functions, which are determined from the boundary conditions (5)–(11d).

Let us define two functions in the following way:

$$V_2(x_1) = \begin{cases} \dfrac{\partial U_2(x_1, 0)}{\partial x_1} & \text{when} \quad x_1 \in (a, b) \\ 0 & \text{when} \quad x_1 \in (0, a) \cup (b, \infty) \end{cases} \tag{16}$$

$$V_1(x_1) = \begin{cases} \dfrac{\partial \varphi(x_1, 0)}{\partial x_1} & \text{when} \quad x_1 \in (a, b) \\ 0 & \text{when} \quad x_1 \in (0, a) \cup (b, \infty) \end{cases} \tag{17}$$

According to the boundary conditions (5), (7), (9) and (10), the following relations are obtained:

$$\alpha A(\alpha) = \int_a^b V_2(t)\sin \alpha t \, \mathrm{d}t; \quad \alpha C_1(\alpha) = \int_a^b V_1(t)\sin \alpha t \, \mathrm{d}t; \quad V_1(t) = \frac{\chi}{\mu_r} V_2(t) \tag{18}$$

$$B(\alpha) = e_1 \int_a^b V_2(t)\sin \alpha t \, \mathrm{d}t; \quad e_1 = \frac{1}{4(1-\nu)}\left[2 + \frac{(3-4\nu)\chi^2 b_c^2}{\mu_r^2}\right]. \tag{19}$$

From the boundary condition (8) the function $A_e(\alpha)$ can be determined via function $V_2(t)$. However, we will not need it hereafter. Now we have to determine the unknown functions $E(\beta)$; $F(\beta)$; $C_2(\eta)$ and $G(\beta)$ through the function $V_2(t)$. Substituting Eqs. (18) and (19) into Eq. (12) and using the boundary conditions (11a)–(11d), upon some transformations for the unknown functions we get the following relations:

$$\beta E(\beta) = \eta_k^E \int_a^b V_2(t)\mathrm{e}^{-\beta t} \, \mathrm{d}t + \theta_k^E \beta \int_a^b t V_2(t)^{-\beta t} \, \mathrm{d}t; \quad \beta C_2(\beta) = \eta_k^C \int_a^b V_2(t)\mathrm{e}^{-\beta t} \, \mathrm{d}t + \theta_k^C \beta \int_a^b t V_2(t)^{-\beta t} \, \mathrm{d}t;$$

$$F(\beta) = \eta_k^F \int_a^b V_2(t)\mathrm{e}^{-\beta t} \, \mathrm{d}t + \theta_k^F \beta \int_a^b t V_2(t)^{-\beta t} \, \mathrm{d}t \tag{20}$$

where

$$\eta_k^E = \left[g_k - \frac{T_k\chi}{\mu_r(\mu_r+1)} + \frac{G_k M_k \chi}{N_k \mu_r(\mu_r+1)} - \frac{G_k m_k}{N_k}\right]\left[1 - \frac{T_k\rho_k}{\mu_r+1} + \frac{G_k M_k \rho_k}{N_k(\mu_r+1)} - \frac{G_k}{N_k}\right]^{-1};$$

$$\theta_k^E = \left[t_k - \frac{G_k n_k}{N_k}\right]\left[1 - \frac{T_k\rho_k}{\mu_r+1} + \frac{G_k M_k \rho_k}{N_k(\mu_r+1)} - \frac{G_k}{N_k}\right]^{-1} \eta_k^F$$

$$= \frac{1}{N_k}\left[m_k - \frac{M_k\chi}{\mu_r(\mu_r+1)}\right] + \frac{\eta_k^E}{N_k}\left[\frac{M_k\rho_k}{\mu_r+1} - 1\right];$$

$$\theta_k^F = \frac{n_k}{N_k} + \frac{\theta_k^E}{N_k}\left[\frac{M_k\rho_k}{\mu_r+1} - 1\right]$$

$$\eta_k^C = -\frac{\rho_k \eta_k^E}{\mu_r+1} + \frac{\chi}{\mu_r(\mu_r+1)}; \quad \theta_k^C = -\frac{\rho_k\theta_k^E}{\mu_r+1}, \quad (k = A, B, C, D). \tag{21}$$

In the case (A) $k = A$ and

$$\rho_A = 0;\ N_A = -3 + 4\nu;\ M_A = -\gamma_1;\ m_A = 1;\ n_A = -e_1;\ G_A = T_A = g_A = t_A = 0.$$

In the case (B) $k = B$ and

$$\rho_B = -\chi;\ N_B = -1 + 2\nu;\ M_B = \frac{2\nu\chi b_c^2}{\mu_r};\ m_B = \frac{\chi^2 b_c^2}{2\mu_r^2};\ n_B = -e_1;\ G_B = -2 + 2\nu;\ T_B = \frac{(4\nu - 1)\chi b_c^2}{2\mu_r};$$

$$g_B = t_B = 0.$$

In the case (C) $k = C$ and

$$\rho_C = \chi;\ N_C = -1 + 2\nu;\ M_C = \frac{2\nu\chi b_c^2}{\mu_r};\ m_C = \frac{\chi^2 b_c^2}{2\mu_r^2};\ n_C = -e_1;\ G_C = -3 + 4\nu;\ T_C = -\gamma_1;\ g_C = 1;$$

$$t_C = -e_1.$$

In the case (D) $k = D$ and

$$\rho_D = 0;\ N_D = -2 + 2\nu;\ M_D = -\frac{(1 - 4\nu)\chi b_c^2}{2\mu_r};\ m_D = n_D = G_D = T_D = g_D = t_D = 0.$$

Finally all the unknown functions are expressed through one unknown function $V_2(t)$. Thus, owing to Eqs. (18) and (19), the boundary conditions (5) and (7)–(11d) are satisfied. From the boundary condition (6), one can determine the unknown function $V_2(t)$. Substituting Eqs. (18) and (19) into Eq. (6), the following equation is obtained

$$\frac{1}{\pi} \cdot \int_a^b V_2(t) \cdot R^0(x_1, t)\, \mathrm{d}t = P - P_0(x_1), \quad a < x_1 < b, \tag{22}$$

where

$$R^0(x_1, t) = a_1 \frac{1}{t - x_1} + a_2^k \frac{1}{t + x_1} + a_3^k \frac{x_1}{(t + x_1)^2} + a_4^k \frac{t}{(t + x_1)^2} + a_5^k \frac{t \cdot x_1}{(t + x_1)^3}$$

$$a_1 = 4\left[-1 + (1 - 2\nu)e_1 + \frac{\chi}{\mu_r}\gamma_1^0\right],\ a_2^k = 4\left[a_1 + \eta_k^E + \eta_k^F \cdot (-3 + 2\nu) + \eta_k^C \cdot \gamma_2^0\right],$$

$$a_3^k = 4\eta_k^F,\ a_4^k = 4\left[\theta_k^E + \theta_k^F(2\nu - 3) + \theta_k^C \cdot \gamma_2^0\right], \quad a_5^k = 8\theta_k^F,\ (k = A, B, C, D)$$

$$\gamma_1^0 = \frac{\chi^2 b_c^2}{2\mu_r} + \frac{(2\nu - 1)\chi b_c^2}{\mu_r};\ \gamma_2^0 = \gamma_1^0 - \frac{\chi^2 b_c^2}{\mu_r},\ P = \frac{\chi(\chi - 2)b_c^2}{2\mu_r^2}$$

Taking into account Eq. (16) and the boundary condition (10), one can obtain

$$\int_a^b V_2(t)\, \mathrm{d}t = U_2(0, b) - U_2(0, a). \tag{23}$$

From the symmetry of the problem with respect to OX_1 it follows, that $U_2(b,0)=0$. When $a\neq 0$, satisfied also $U_2(a,0)=0$, and from Eq. (23) we have

$$\int_a^b V_2(t)\,\mathrm{d}t = U_2(0,b) - U_2(0,a) = 0. \tag{24}$$

Thus for the determination of $V_2(t)$ (in the case $a\neq 0$) the singular integral Eq. (22) is obtained together with the boundary condition (24). In the same way, one can consider the case when the inner crack reaches the boundary of the half-plane (case of the edge crack, i.e. when $a=0$). In this case also all formulae (1)–(22) are valid and the kernel $R^0(x_1,t)$, besides the ordinary singularities when $x_1=t$, has a point singularity, when $x_1=t=0$. Note that in cases (A) and (C) from boundary conditions (11a) and (11c) it follows, that $U_2(0,0)=0$, and the condition (24) is obtained for the function $V_2(t)$. In the cases (B) and (D) the condition (24) does not hold. However, the following condition must be satisfied $U_2(0,0)\neq 0$ (see Panasuk et al., 1976).

Taking into account Eqs. (18) and (19) one can obtain from Eq. (12)

$$\frac{t^s_{ij}(x_1,x_2)}{\mu} = \frac{t_{ij}(x_1,x_2)}{\mu} + \frac{t^M_{ij}(x_1,x_2)}{\mu} = \frac{(4\mu_r-3)b_c^2}{2\mu_r^2}\delta_{ij} + \frac{4}{\pi}\int_a^b R_{ij}(x_1,x_2,t)V_2(t)\,\mathrm{d}t \quad (i,j=1,2) \tag{25}$$

where for example

$$\begin{aligned} R_{22}(x_1,x_2,t) = {} & \frac{1}{2}\left[-1+(1-2\nu)e_1+\frac{\chi}{\mu_r}\gamma_1''\right]\left[\frac{t+x_1}{(t+x_1)^2+x_2^2}+\frac{t-x_1}{(t-x_1)^2+x_2^2}\right] - \frac{e_1}{2}\left[\frac{2(t+x_1)x_2^2}{\left[(t+x_1)^2+x_2^2\right]^2}\right. \\ & \left. + \frac{2(t-x_1)x_2^2}{\left[(t-x_1)^2+x_2^2\right]^2}\right] + \left[\eta_k^E+\eta_k^F(2\nu-3)+\gamma_2''\eta_k^C\right]\frac{t+x_1}{(t+x_1)^2+x_2^2} + \eta_k^F\frac{x_1\left[(t+x_1)^2-x_2^2\right]}{\left[(t+x_1)^2+x_2^2\right]^2} \\ & + \left[\theta_k^E+\theta_k^F(2\nu-3)+\gamma_2''\theta_k^C\right]\frac{t\left[(t+x_1)^2-x_2^2\right]}{\left[(t+x_1)^2+x_2^2\right]^2} + \theta_k^F\frac{2tx_1(t+x_1)\left[(t+x_1)^2-3x_2^2\right]}{\left[(t+x_1)^2+x_2^2\right]^3}, \end{aligned}$$

$$\gamma_1'' = \frac{2(2\nu-1)\chi b_c^2}{\mu_r} - \frac{2\mu_r-1}{\mu_r}b_c^2,\quad \gamma_2'' = \gamma_1'' + \frac{2(2\mu_r-1)}{\mu_r}b_c^2.$$

In particular cases, from Eq. (22) one can obtain well-known results. For example, in case (B) when $b_c^2=0$ (pure elastic case), the integral Eq. (22) takes the form

$$\int_a^b\left[\frac{1}{t-x_1}+\frac{1}{t+x_1}+\frac{2t}{(t+x_1)^2}-\frac{4t^2}{(t+x_1)^3}\right]V_2(t)\,\mathrm{d}t = \pi P_0(x_1),$$

which coincides with the integral equations, obtained by Panasuk et al. (1976) and Nied (1987). In the case (A) when $b_c^2=0$, the integral equation takes the form of that, obtained by Bereznicki et al. (1983).

3. Construction of the solution to the integral equation

First, the case of an inner crack is considered (i.e. $a\neq 0$). Let us write the integral Eq. (22) in the from

$$\int_{-1}^{1} \overline{V_2}(s)\left[\frac{a_1}{s-r} + L(r,s)\right] \mathrm{d}s = \pi(P - P_0(r)), \quad |r| < 1 \tag{26}$$

where the following notations are introduced

$$x_1 = \frac{b-a}{2}(r+d),\ t = \frac{b-a}{2}(s+d),\ d = \frac{b+a}{b-a},\ \overline{V_2}(s) = V_2(t),\ L(r,s)$$
$$= \frac{a_2^k}{s+r+2d} + \frac{a_3^k(r+d)}{(s+r+2d)^2} + \frac{a_4^k(s+d)}{(s+r+2d)^2} + \frac{a_5^k(r+d)(s+d)}{(s+r+2d)^3}$$

The condition (24) is written in the form

$$\int_{-1}^{1} \overline{V_2}(t)\, \mathrm{d}t = 0 \tag{27}$$

In the cases of $a \neq 0$ the integral Eq. (26) must be considered together with the condition (27).

In the case of an edge crack (i.e. $a = 0$ or $d = 1$), for determination of the unknown function $\overline{V_2}(s)$, also the integral Eq. (26) is obtained. When $a = 0$, in the case (A) and (C) the integral Eq. (26) is considered together with the condition (27).

Note that on the basis of Eqs. (26) and (27), the solution of the problem for an infinite plane with a crack can be obtained (see Shindo, 1977). In fact, considering$d \to \infty$ one gets from Eq. (26)

$$\frac{1}{\pi}\int_{-1}^{1} \frac{\overline{V_2}(s)}{s-r}\, \mathrm{d}s = \frac{P - P_0(r)}{a_1}, \quad |r| < 1.$$

The solution of this equation tending to ∞ when$r \to \pm 1$ takes the form

$$\overline{V_2}(s) = \frac{P - P_0}{a_1}\frac{s}{\sqrt{1-s^2}}.$$

The solution is received for $P_0(r) = P_0 = \text{const}$.

The coefficient of intensity of magnetoelastic stress $t_{22}^s(x_1, x_2)/\mu$ in this case takes the form:

$$k^\infty = \lim_{x_1 \to +\infty} \sqrt{2l(x_1 - 1)}\frac{t_{22}^s(x_1, 0)}{\mu} = -\frac{l^{1/2}(P - P_0)}{1 - (1-\nu)\mu_r b_c^2},$$

which coincides with (Shindo's, 1977) result (with accuracy $1/\mu_r \ll 1$, and $P_0 = 0$), obtained by another method.

Let us represent the function $\overline{V_2}(s)$ in the form

$$\overline{V_2}(s) = u(s)\, \omega(s), \tag{28}$$

where $u(s)$ is the new regular unknown function and $\omega(s) = (1-s)^{-\alpha}(1+s)^{-\beta}$. Using the results of Erdogan et al. (1973) in the case of an inner crack ($a \neq 0$) we can take $\alpha = \beta = 0.5$. In the case of an edge crack $a = 0$), for determination of α and β we obtain

$$\begin{cases} \alpha = 0.5 \\ \cos \pi\beta - 0.5 a_5^k \beta^2 + \left(a_4^k - a_3^k - 0.5 a_5^k\right)\beta + a_2^k + a_4^k = 0 \end{cases} \tag{29}$$

We can see from the numerical calculations, that the solution β of transcendental equation in the case (A) is less than 0.5 when $b_c^2 < b_{c0}^2$ (for example, when $\nu = 0.3$, $\mu_r = 10^4$, $b_{c0}^2 = 0.0000233$). The solution

Table 1
Solution of Eq. (29) in the case (A) for edge crack ($a = 0$), when $\mu_r = 10^4$, $\nu = 0.3$

$b_c^2 \times 10^5$	0	1/3	2/3	1	4/3	5/3	2	7/3	8/3	3
β	0.385	0.403	0.419	0.435	0.452	0.467	0.482	0.498	0.512	0.527

of the Eq. (29) with the account of the condition $0 < \mathrm{Re}\,\alpha$, $\mathrm{Re}\,\beta < 1$ in the case (A) is given in Table 1. Notice that the Eq. (29) has no solution satisfying the condition $0 < \mathrm{Re}\,\beta < 1$ in the cases (B), (C) and (D). From the given table it follows, that in the case of inner crack ($a \neq 0$) one may take $\alpha = \beta = 0.5$ for all considered boundary conditions (i.e. in the cases (A), (B), (C) and (D)). When $a = 0$, one may take $\alpha = \beta = 0.5$ in the cases (B), (C) and (D). In the case (A) α and β can be taken $\alpha = \beta = 0.5$ too. (But in difference to the other cases, here the condition $b_c^2 < b_{c0}^2$ must be satisfied.

For solving the Eq. (26), we seek $u(s)$ in the form of interpolation polynomial (see Erdogan et al., 1973; Panasuk et al., 1976)

$$u(s) = \frac{2}{M}\sum_{m=1}^{M} u(t_m)\sum_{r=0}^{M-1} T_r(t_m)T(s) - \frac{1}{M}\sum_{m=1}^{M} u(t_m), \tag{30}$$

where $T_n(x)$ is a Chebyshev polynomial of the first kind. Using the quadrant formulae

$$\int_{-1}^{1} \frac{(1-t^2)^{-0.5} u(t)\,\mathrm{d}t}{t - x_k} = \sum_{m=1}^{M} \frac{\pi}{M}\frac{u(t_m)}{t_m - x_k}, \quad \int_{-1}^{1} (1-t^2)^{-0.5} u(t)\,\mathrm{d}t = \sum_{m=1}^{M} \frac{\pi}{M} u(t_m), \tag{31}$$

from the integral Eq. (26) we obtain the system of $M - 1$ linear algebraic equations for determination of M unknown constants $u(t_m)$ ($m = 1, 2, \ldots, M$)

$$\frac{1}{M}\sum_{m=1}^{M} u(t_m)\left[\frac{a_1^k}{t_m - x_n} + L(t_m, x_n)\right] = \pi\big(P - P_0(x_n)\big), \quad (n = 1, 2, \ldots, M-1) \tag{32}$$

where$t_m = \cos(\frac{2m-1}{2M}\pi)$; $x_n = \cos(\frac{\pi n}{M})$. For the inner crack ($a \neq 0$) the condition (27) with the account of Eq. (31) can be written in the form

$$\sum_{m=1}^{M} \frac{\pi}{M} u(t_m) = 0 \tag{33}$$

For the edge crack ($a = 0$), in the cases (A) and (C) the numerical method is similar to the case of an inner crack ($a \neq 0$), i.e. for determination of $u(t_k)$ the Eqs. (32) and (33) have to be solved. The numerical calculations prove, that the function $\overline{V_2}(s)$ at the point $s = -1$ (what corresponds to $x_1 = 0$) has singularity of less order than $(1 + s)^{-0.5}$, which yields the following condition in cases (B) and (D) (see Panasuk et al., 1976)

$$u(-1) = 0. \tag{34}$$

From Eq. (30) the condition (34) can be written in the form

$$\sum_{m=1}^{M} (-1)^m u(t_m)\,\mathrm{tg}\left(\frac{2m-1}{4M}\pi\right) = 0. \tag{35}$$

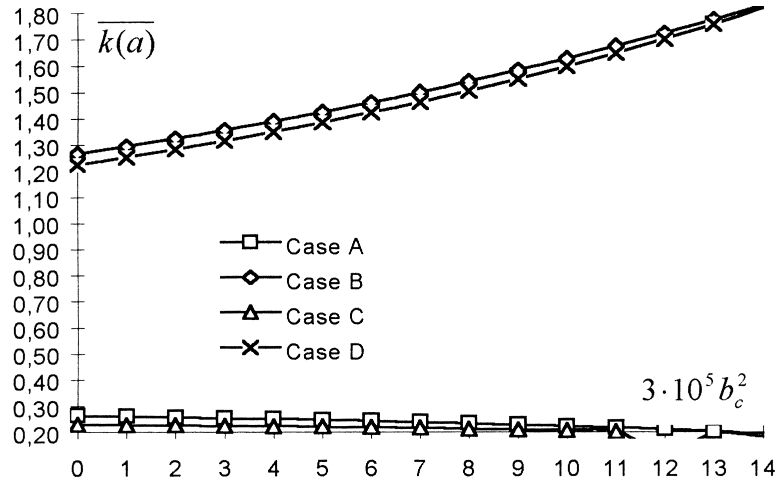

Fig. 2. Magnetoelastic stress intensity factors at crack tip a subjected in a magnetic field, for cases (A), (B), (C), and (D) when $\mu_r = 10^4$, $\nu = 0.3$, $d = 1.05$.

Thus, the Eq. (32) also has to be solved with the account of condition (35) for the edge crack in the cases (B) and (D). Upon solution of the system of algebraic Eqs. (32) and (33) or Eqs. (32) and (35), from formula (30) we obtain $u(s)$ and for the coefficient of intensity of magnetoelastic stresses, with the account of (25), we have (when $P_0(r) = P_0 = \text{const}$)

$$k(a) = \sqrt{l} \lim_{r \to -1+0} \sqrt{r+1}\, t_{22}^s(r,0) = 4\sqrt{l}(P_0 - P) b_1 u(-1) = k^\infty a_1 u(-1)$$

$$k(b) = \sqrt{l} \lim_{r \to 1+0} \sqrt{r-1}\, t_{22}^s(r,0) = 4\sqrt{l}(P_0 - P) b_1 u(1) = k^\infty a_1 u(1) \tag{36}$$

where $b_1 = 0.5\{-1 + (1-2\nu)e_1 + \frac{\chi b_c^2}{\mu_r^2}[4\mu_r(\nu - 1) + 3 - 4\nu]\}$. From Eq. (30) we have

$$u(1) = -\frac{1}{M}\sum_{m=1}^{M}(-1)^{m+1} u(t_m)\,\mathrm{ctg}\left(\frac{2m-1}{4M}\pi\right)$$

$$u(-1) = \frac{1}{M}\sum_{m=1}^{M}(-1)^{m+M} u(t_m)\,\mathrm{tg}\left(\frac{2m-1}{4M}\pi\right) \tag{37}$$

Note that the formula (35) is obtained with the account of the following relations:

$$\frac{1}{\pi}\int_{-1}^{1} \frac{T_k(s)}{\sqrt{1-s^2}} \frac{\mathrm{d}s}{s-t} = \begin{cases} U_{k-1}(t) \text{ when } |t| < 1 \\ \dfrac{1}{\sqrt{t^2-1}}\left(t + \sqrt{t^2-1}\right)^k \text{ when } t < -1 \\ -\dfrac{1}{\sqrt{t^2-1}}\left(t - \sqrt{t^2-1}\right)^k \text{ when } t > 1 \end{cases}. \tag{38}$$

From Eq. (26) it follows that a fredholm integral equation is obtained when $b_c^2 = b_{ccr}^2$ (b_{ccr}^2 is a solution

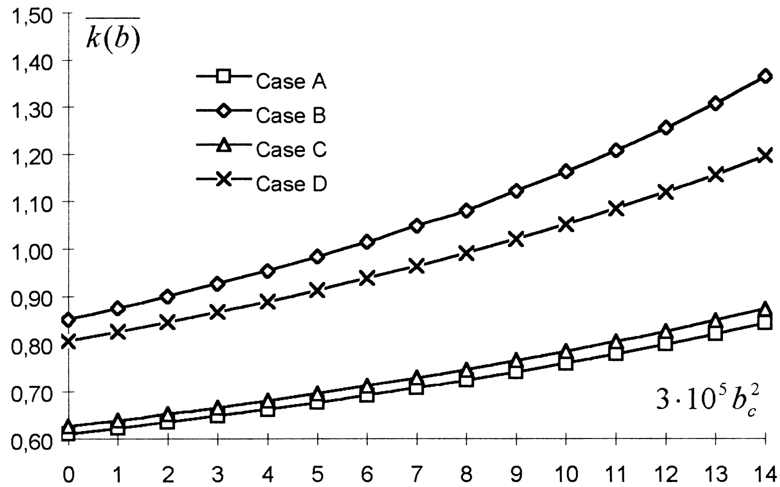

Fig. 3. Magnetoelastic stress intensity factors at crack tip b subjected in a magnetic field, for cases (A), (B), (C), and (D) when $\mu_r = 10^4$, $\nu = 0.3$, $d = 1.05$.

of equation $a_1 = 0$). In that case $\overline{V}_2(t)$ has no singularities at $t = \pm 1$. The intensity coefficient of magnetoelastic stresses at $b_c^2 = b_{ccr}^2$ is equal to zero. When $d \to \infty$ (infinite plane with crack) it is clear (see Shindo, 1977), that the coefficient of magnetoelastic stresses tends to infinity when $b_c^2 \to b_{ccr}^2 = 2\mu_r^2 / 2\chi^2[2\chi(1-\nu) - 1 + 2\nu]$. However, the account of the boundary of the body yields the following result: for $b_c^2 \to b_{ccr}^2$ the intensity coefficient tends to zero.

4. Numerical results

On the basis of the analysis and the numerical procedure presented in previous sections, a computer program is developed and numerical results are obtained. For large values of M ($M \approx 60$) the

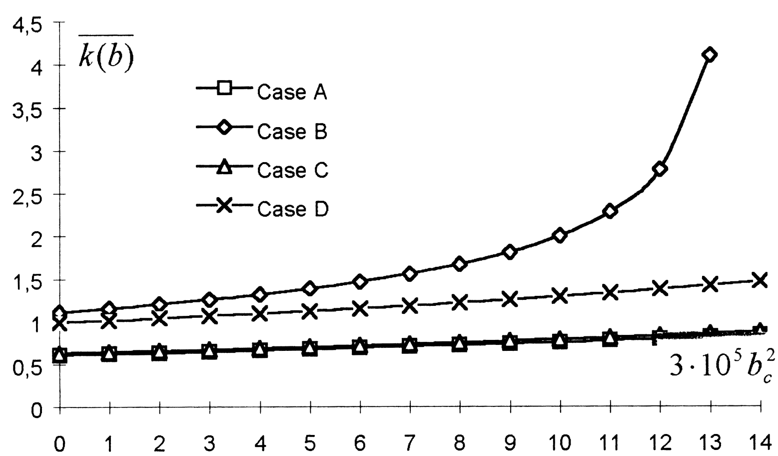

Fig. 4. Magnetoelastic stress intensity factors for an edge crack, subjected in a magnetic field when $\mu_r = 10^4$, $\nu = 0.3$.

Table 2
Depending magnetoelastic stress intensity factors for a crack in the case (A), (B), and (C) and (D) depending upon d, when $\mu_r = 10^4$, $\nu = 0.3$, $b_c^2 = 0.00004$

d	1.005	1.5	2.0	2.5	3.0	3.5	4.0
k^{A}(b)	0.795	0.835	0.869	0.894	0.911	0.923	0.932
k^{A}(a)	0.083	0.601	0.744	0.813	0.851	0.875	0.891
k^{B}(b)	1.389	1.082	1.039	1.019	1.007	0.999	0.994
k^{B}(a)	2.315	1.179	1.069	1.023	0.999	0.985	0.976
k^{C}(b)	0.830	0.838	0.870	0.894	0.911	0.923	0.933
k^{C}(a)	0.057	0.603	0.746	0.814	0.853	0.876	0.892
k^{D}(b)	1.191	1.022	0.999	0.989	0.984	0.981	0.979
k^{D}(a)	2.263	1.051	0.991	0.971	0.962	0.957	0.969

determinant of Eqs. (32) and (33) or Eqs. (32) and (35) is of order $O(10^{-25})$, and numerical calculation is complicated, although good converging results are obtained starting from $M \approx 40$. In Figs. 2–4 and in Table 2 the numerical values of $\overline{k(a)}(a) = k(a)/k^{\infty}$ and $\overline{k(b)} = k(b)/k^{\infty}$ are given for cases (A), (B), (C) and (D). Numerical results are given for the inner crack ($a \neq 0$) and for the edge crack ($a = 0$). In all calculations $P_0 = 0.001$.

4.1. Numerical results for the inner crack ($a \neq 0$)

From numerical calculations one can see, that in all the cases with the growth of d the $\bar{k}(a)$ and $\bar{k}(b)$ tend to 1 (particularly when $d \to \infty$ the result of Shindo (1977) is obtained). In Table 2 the values of the intensity coefficients $\bar{k}(a)$ and $\bar{k}(b)$ are given depending upon d under different surface conditions when $\mu_r = 10^4$, $\nu = 0.3$, $b_c^2 = 0.00004$. Numerical analysis has shown, that boundary conditions do not influence much (compared with the case of a plane with a crack) the stress–strain state and on the coefficient of intensity of magnetoelastic stresses when $d > 5$. In Figs. 2 and 3 the graph of intensity coefficient of magnetic stress depending upon magnetic field for inner crack is given.

The value b_c^2 increases up to 4.67×10^{-5} and $\mu_r = 10^4$, $\nu = 0.3$, $d = 1.05$. From numerical calculations one can see:

- In cases (A) and (C) the value $\bar{k}(a)$ decreases with the increase of b_c^2;
- In cases (B) and (D) the $\bar{k}(a)$ increases with the increase of b_c^2;
- With the increase of b_c^2 the value $\bar{k}(b)$ is increasing in all cases;
- When $b_c^2 \to b_{ccr}^2$ (b_{ccr}^2 is solution of $a_1 = 0$) the values $\bar{k}(a)$ and $\bar{k}(b) \to 0$;
- When $\mu_r < 5000$ the values $\bar{k}(a)$ and $\bar{k}(b)$ change slightly depending upon magnetic field (about 3%). Great changes take place when $\mu_r > 8000$ for rather weak magnetic field ($\sim$1 T);
- Divergence is rather strong between intensity coefficient in case (B) and (D) on the right side of the crack (when $x_1 = b$);
- For the coefficients of intensity the following comparison relations are obtained: $\bar{k}^C(a) < \bar{k}^A(a) < \bar{k}^D(a) < \bar{k}^B(a)$ and $\bar{k}^A(b) < \bar{k}^C(b) < \bar{k}^D(b) < \bar{k}^B(b)$ (where $\bar{k}^{A,\,B,\,C,\,D}$ are the coefficients of intensity in the case (A), (B), (C) and (D), respectively).

4.2. Numerical results in the case of edge crack ($a = 0$)

In Fig. 4 the graph of intensity coefficient of magnetoelastic stresses $\bar{k}(b)$ depending upon magnetic field b_c^2 is presented. The following parameters are chosen for calculations: $\mu_r = 10^4$, $\nu = 0.3$ and b_c^2 varies from 0 up to 4.67×10^{-5}. The numerical calculations show, that here also the same pattern is fixed, as in the case of the inner crack. Divergence between intensity coefficients in cases (B) and (D) increases with the increase of the magnetic field. From the analysis one can conclude, that intensity coefficient has the same value in the cases (A) and (C). In the case (A) (when $b_c^2 = 0$) from numerical results one can get $\bar{k}(b) = 0.618$ (see Fig. 4). That result coincides with the similar result obtained by Savruk (1988). In the case (B) $\bar{k}(b) = 1.1215$, which has been obtained by Koiter (1965), Panasuk et al. (1976) and Savruk (1988).

5. Conclusions

Thus the problem of determination of the stress–strain state for a magnetoelastic half-plane with a crack is solved. It is supposed that four different boundary conditions are given on the boundary of the half-plane. It is shown, that for determination of the stress–strain state a singular integral equation must be obtained. A numerical algorithm is developed for solving the integral equation. It has been shown that the boundary conditions essentially influence the coefficients of intensity and magnetoelastic stresses when $d < 5$. It is shown that, the coefficient of intensity increases with the increase of the magnetic field. Starting from a certain value of a magnetic field it decreases tending to 0 (whereas in the case of infinite plane the coefficient of intensity increases up to infinity).

In the case of the edge crack ($a = 0$) an integral equation can be obtained with a difference kernel on semi-infinite segment. This problem can be handled analytically by Wiener–Hopf's method. As a result, the coefficients of intensity and magnetoelastic stresses might be obtained analytically. However, this problem is out of the scope of this paper.

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